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Shape of the Plasma Boundary in TBR

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Abstract A diagnostic system to determine the shape of the cross-section of the plasma column in TBR-1 has been developed. The system relies on measurements of radial and azimuthal components of the magnetic field around the plasma column and is based upon a technique developed by Swain and Neilson. It is shown that during normal discharges the plasma column undergoes a systematic downward shift and shrinks in minor and major radii.

1. INTRODUCTION

The basic equilibrium characteristics of tokamak discharges are determined through measurements of the toroidal plasma current, toroidal and vertical components of the external magnetic field, and loop voltage. However, more detailed information is needed to study the magnetohydrodynamic stability conditionsof the discharges. In this case, it is necessary to determine the shape of the cross section of the plasma column and the radial profiles of the pressure and current density. Usually, the pressure profile is inferred from measurements of the electron temperature profile through Thompson scattering and of plasma density through microwave or laser interferometry. The current profile can at present be only indirectly determined.

Many different methods have been developed in the lastdecadeto determine the shape of the cross section of the plasma column. Although all the methods rely on measurements of the magnetic flux and/or components of the poloidal magnetic field outside the plasma column, they substantially vary in the degree of sophistication to model the current distribution in the plasma. The more accurate methods solve the free-boundary magnetohydrodynamic equilibrium problem and adjust the shape

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of the cross section and other free parameters to reproduce external measurements and the pressure profile. Because these methods self-consistently solve the equilibrium problem, they allow not only the determination of the shape of the plasma cross section, but also an indirect determination of the poloidal beta

\[ \beta_p = \frac{1}{2\mu_0} \frac{\int_p B^2 \, dV}{\frac{1}{V} \langle B^2 \rangle}, \tag{1} \]

the profile of the safety factor,

\[ q = B_{TO} R_0 \int_\psi \frac{d\sigma}{R^2 B_p} \tag{2} \]

and the internal inductance of the plasma column,

\[ \ell_i = \frac{1}{V} \int p B^2 \, dV \langle B^2 \rangle \]

In these expressions \( R \) is the radial coordinate of a point on a flux surface, \( R_0 \) is the major radius of the geometric center of the plasma column, \( B_p \) is the poloidal component of the magnetic field, \( B_{TO} \) is the toroidal magnetic field at \( R = R_0 \), \( p \) is the plasma pressure, \( V \) is the plasma volume, \( d\sigma \) is the element of area length along the poloidal direction of a \( \psi = \text{const} \) flux surface, and \( \langle \rangle \) represents the average over the plasma cross section.

The major drawback of methods based upon a self-consistent solution of the magnetohydrodynamic equilibrium equation is that they take a substantial amount of computer time. Although the knowledge of \( \beta_p \) and \( q \) profile is essential for a stability analysis, an experimentalist usually needs quick information about the value of \( \beta_p \) and the shape of the cross section to properly adjust external parameters between discharges. For this reason, less accurate, swifter methods have also been developed. These methods are based upon modelling the toroidal plasma current by a set of discrete filaments whose positions are a priori fixed. The currents in these filaments are calculated from the minimization of the average square deviation of measured and calculated values of the magnetic field at the positions of diagnostic
probes. In particular, Swain and Neilson\textsuperscript{6} have developed a technique that allows a very fast determination of the shape of the cross section of the plasma column and an accurate estimate for the Shafranov parameter\textsuperscript{11}
\[ \Lambda = \beta_p + \frac{\ell^2}{2}. \] (4)

In this paper we report the results of measuring the shape of the cross section of the plasma column in the TBR-1 tokamak\textsuperscript{12}. Previous measurements of resistive modes in TBR-1 have shown a systematic downward displacement of the mode structure\textsuperscript{13,14}. It was not clear whether this displacement occurred only in the perturbation or was caused by a displacement of the equilibrium position of the plasma column as a whole. The measurements carried out with the experimental apparatus described in this paper have found that the plasma column not only shifts downwards during the discharges but also shrinks and increases the internal inductance.

In the next section we present a summary of the technique developed by Swain and Neilson. In section 3 we describe the diagnostic system that has been developed at the Instituto de Física of Universidade de São Paulo. In section 4 we present the main results and the conclusion.

2. ANALYTICAL MODEL

The equilibrium of the plasma column in a tokamak is maintained by the poloidal field created by currents that circulate in the plasma and in external coils\textsuperscript{15}. The latter produce a magnetic field that in the plasma region is mainly parallel to the symmetry axis of the torus and thus are called vertical field coils. The plasma current, on the other hand, is induced inside the vacuum chamber by the time variation of a magnetic flux through the central hole of the torus. The coils that produce this flux are part of the so-called ohmic heating transformer (OHT)\textsuperscript{16}. In fig. 1 we show a toroidal cross section of the TBR-1 tokamak showing the vertical field coils, the OHT coils, the vacuum chamber, and the edge of the current limiter. The limiter is a metal ring that limits the current channel and keeps the hot plasma away from the vacuum chamber.
Fig. 1 - Cross section of the TBR-I tokamak showing the vacuum chamber (a), the edge of the current limiter (b), the vertical field coils (c), the ohmic heating coils (d), and the filaments to simulate the plasma current (e).

Let us assume that outside the current channel, in the shadow of the limiter, we put a set of N pick-up coils that measure the radial and azimuthal components of the magnetic field during a discharge. This field is produced by the plasma current and by the currents in all external coils, which are known. Although the total plasma current can be measured, the profile of the corresponding current density and the shape of the current channel are unknown. The technique of Whiten and Neilson⁶ consists in first simulating the plasma current
density by a set of M toroidal filaments located at pre-chosen positions. We shall refer to those filaments as plasma filaments; four of them are indicated in fig. 1. Let us denote the current in the external coils by \( I_j^e \) and the currents in the plasma filaments by \( I_j^p \). The index \( j \) runs from 1 to the total number \( J \) of external coils in the first case and from 1 to \( M \) in the second case. The unknown currents \( I_j^p \) are then determined by minimizing the average square deviation of the magnetic field

\[
\langle \Delta B^2 \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{(B_{i,cal} - B_{i,m})^2}{\sigma_i^2},
\]

where \( B_{i,cal} \) and \( B_{i,m} \) are respectively the calculated and measured values of the poloidal (radial and azimuthal components) magnetic field and \( \sigma_i \) is the estimated standard deviation reflecting the measurement uncertainty at sensor \( i \). It can be shown that the minimization of \( \langle \Delta B^2 \rangle \) leads to a system of \( M \) linear algebraic equations for the unknown currents

\[
I_j^e \sum_{i=1}^{N} \frac{B_{i,m} - B_{i,e}}{\sigma_i^2} \phi_{i,j}^p = \sum_{j=1}^{M} I_j^p \sum_{k=1}^{M} \frac{\phi_{j,k}^p}{\sigma_i^2}, \quad k = 1, 2, \ldots, M,
\]

where \( B_{i,e} \) is the total field at position \( i \) produced by the currents in the external filaments,

\[
B_{i,e} = \sum_{j=1}^{J} \phi_{i,j}^p I_j^e,
\]

and \( \phi_{\alpha\beta}^p \) are matrices corresponding to Green's functions, i.e., they give the magnetic field produced at position \( \alpha \) by a unit current located at position \( \beta \). In the present case, all the external coils and plasma filaments are concentric circular loops and the expressions for \( \phi_{\alpha\beta}^p \) can be readily obtained. Since the total plasma current \( I_p \) is known, the minimization of \( \langle \Delta B^2 \rangle \) can be carried out imposing the constant

\[
\sum_{j=1}^{M} I_j^p = I_p.
\]

However, this constraint practically does not change the results obtained without imposing it. Actually this occurs because the currents...
\( \mathcal{I}_J^P \) in the plasma filaments have no physical meaning. They only approximately give the magnetic field produced by the real plasma current outside the current channel and their first moments correspond to the first moments of the current profile in the plasma\(^6\).

Once the currents \( \mathcal{I}_J^P \) are calculated, one can find the flux function  \( \psi(R,Z) = 2\pi R A_\phi \), where \((R,\phi,Z)\) are cylindrical coordinates centred at the symmetry axis of the torus and \( A_\phi \) is the toroidal component of the vector potential\(^19\). Since the plasma boundary has to be a flux surface, to determine the cross section of the plasma column one has to find the flux surface that touches the limiter in just one point. This is carried out by a numerical code that directly plots the plasma cross section\(^17\).

Two important equilibrium quantities can be calculated from the knowledge of the currents \( \mathcal{I}_J^P \) in the plasma filaments and the shape of the cross section of the plasma column. The first quantity is the radial displacement of the magnetic axis with respect to the geometric center of the plasma cross section, the so-called Shafranov shift\(^15\). This is given by

\[
\Delta = \frac{1}{\mu_0 \mathcal{I}_P} \oint_c \left( -\xi B_t + Z B_n \right) d\ell
\]

where \( \xi = R - R_0 \) and \( B_t \) and \( B_n \) are respectively the tangential and normal components of the magnetic field along a closed poloidal contour \( c \) around the plasma column\(^2\). In our case, we choose the contour \( c \) to coincide with the edge of the current limiter because along this contour the magnetic field produced by the currents in the plasma filaments very closely reproduces the field due to the real current distribution\(^6\). Equation (8) is derived using a multipole expansion of the plasma current distribution which assumes up-down symmetry with respect to the equatorial plane of the plasma column\(^2\). In the case of TBR-1, the plasma column is displaced downwards and asymmetric with respect to the quatorial plane. This introduces an additional term into the right-hand side of the above expression for \( \Delta \). Assuming that the downward displacement of the plasma column \( Z_0 \) is much smaller than the major radius, the correction term is to lowest order given by
We have calculated this term for TBR-1 discharges and verified that it introduces a correction smaller than 5% in the final value of $A$; this correction is of the order of the accuracy of the measurements and thus it has not been taken into account.

The other important equilibrium quantity is the Shafranov parameter $A$ defined in eq. (4). This is calculated using the formula

$$A = \frac{s_1}{2} + s_2$$

(9)

where $s_1$ and $s_2$ are two integrals over the plasma surface defined by

$$s_1 = \frac{1}{\mu_0 R I_p^2} \int B_t^2 \rho \hat{n} \cdot \hat{e}_\rho \, dS_n$$

(10)

and

$$s_2 = \frac{1}{\mu_0 I_p^2} \int B_t^2 \hat{n} \cdot \hat{e}_R \, dS_n$$

(11)

where $\rho$ is the radial variable of a pseudo-toroidal coordinate system centred at the magnetic axis of the plasma column and $\hat{e}_\rho$ and $\hat{e}_R$ are unit vectors in the directions $\rho = \text{const.}$ and $R = \text{const.}$, respectively. The integrals appearing in eqs. (8), (10), and (11) are calculated using a numerical code described in reference 17.

To test the accuracy of the method and to determine the optimal number $M$ of plasma filaments, we have carried out numerical simulations of real plasma discharges assuming a parabolic current density profile and using twenty diagnostic coils, corresponding to the actual diagnostic system. The field produced at the position of each diagnostic coil has been calculated and multiplied by a random factor between 0.9 and 1.1 to simulate experimental errors in real measurements. We have found out that a very good approximation to the exact shape of the plasma cross-section can be obtained using three to six plasma filaments ($3 \leq M \leq 6$). Details of the simulations are given in reference 17.
Finally, we have slightly improved the technique of Swain and Neilson to better accommodate the displacement of the plasma column in TER-1. The positions of the plasma filaments are initially chosen with their geometric center coinciding with the one of the vacuum chamber. If the geometric center of the plasma cross section is found to be shifted with respect to the center of the vacuum chamber, the center of the plasma filaments are shifted to that position and the whole calculation repeated. This process goes on until the displacement between two subsequent positions of the center of the plasma cross section is smaller than 0.5 mm. In practice we find that no more than two iterations are sufficient to satisfy this criterion.

3. DIAGNOSTIC SYSTEM

The TBR-1 tokamak is a small-size device that has been designed and constructed at the Instituto de Física of Universidade de São Paulo. Reproducible discharges are obtained after approximately five hours of discharge or radio-frequency cleaning. The plasma current can be varied from 6 to 12 kA, with pulse duration up to 7 ms, and the toroidal magnetic field can be varied from 4 to 5 kG. The vacuum chamber (316 LSS) is 3 mm thick, which gives an attenuation of approximately 13 db for signals in the kHz range. Therefore, we have designed the diagnostic system with the pick-up coils inside the vacuum chamber to prevent attenuation and distortion of the signals.

Two types of coils are used: the radial coils that measure the radial component of the poloidal magnetic field and the tangential coils that measure the azimuthal component. The tangential coils are made of 56 turns of 0.13 mm copper wire directly wound on two 12.7 mm nylon tubes. The radial coils are made of 130 turns of the same wire wound on small spools that are then inserted in transversal orifices in the nylon tubes. The size of the coils and the assembling scheme are indicated in Fig. 2a. The two nylon tubes are then covered with heat-shrink isolation, bent in semicircles of 9.7 mm radius (Fig. 2b), and inserted into stainless steel semicircular tubes of 19 mm internal diameter and 0.3 mm thickness. One end of these supporting tubes is closed with a vacuum tight metal plug and the other end is soldered to a straight
tube that can slide inside a feedthrough in the vacuum flange (fig. 3).

During discharge cleaning, the vacuum chamber and metal surfaces inside it get very hot. In order to avoid melting of the isolation of the pick-up coils, cooling air is forced into the nylon tube and returns through the space between the nylon and supporting tubes, as indicated in fig. 3b. A picture of the entire assembly is shown in fig. 4. This type of construction considerably facilitates the insertion of the diagnostic system into the vacuum chamber, in spite of the small size of the diagnostic ports of TBR-I. By rotating the supporting tubes, the two semi-circles can be brought close together and then inserted into the vacuum chamber. Once the flange is in place, the two tubes are rotated back to the proper positions.

The effective areas of the pick-up coils have been determined...
in a standard Helmholtz coil fed with a 60 Hz cw current. The measured values are given in table 1. In the actual experiment, the signal of each coil has to be integrated. We have designed a very versatile active integrator that can operate either as differential or unipolar amplifier\(^\text{17}\). There is one integrator for each coil. The signals from the integrators are digitalized in Le Croy 2264 modules and stored in Le Croy 8800A and 8800/8 memories for later displaying in an oscilloscope or further computer processing. The reading rate of the modules is fixed at 2.5 μs. The gains of the integrators have been calibrated using a known input signal and a 50Ω load, corresponding to the input impedance of the CAMAC modules. The actual gains used for each coil are shown in table 1.
The toroidal magnetic field in TBR-1 has a strong ripple due to the discreteness of the toroidal field coils. This produces a radial magnetic field component that is detected by the radial pick-up coils. The corresponding signal is so intense that it completely saturates the CAMAC modules. To eliminate this effect, we have added a constant dc voltage to the output of the active integrators that are connected to the radial diagnostic coils. Furthermore, we correct the signal of these coils by subtracting out the spurious signal that is measured in the absence of plasma, by pulsing only the toroidal field coils.

4. RESULTS

We have analysed various discharges in TBR-1 with the plasma current varying from 5.5 to 10.5 kA and with a discharge duration of approximately 6.5 ms. In this paper we report the results obtained for two discharges, one with a current of 8.7 kA and the other with a current of 10 kA. These discharges are typical within the range of parameters that have been used. Discharges with currents smaller than 9 kA usually have a low magnetohydrodynamic activity whereas discharges with currents above 10 kA have strong activity.
Table 1 - Effective area of the pick-up coils and gain of the integrators used in each coil. The labels of the coils correspond to the labels used in fig. 3a.

<table>
<thead>
<tr>
<th>Radial coil</th>
<th>Effective area ( (10^{-3} \text{m}^2) ) (±0.08)</th>
<th>Gain ( (s^{-1}) ) ( \frac{V_{\text{out}}}{\int V_{\text{in}} dt} )</th>
<th>Tangential coil</th>
<th>Effective area ( (10^{-3} \text{m}^2) ) (±0.08)</th>
<th>Gain ( (s^{-1}) ) ( \frac{V_{\text{out}}}{\int V_{\text{in}} dt} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>5.31</td>
<td>3.98</td>
<td>Ti</td>
<td>5.55</td>
<td>817</td>
</tr>
<tr>
<td>R2</td>
<td>5.59</td>
<td>4.06</td>
<td>T2</td>
<td>5.97</td>
<td>707</td>
</tr>
<tr>
<td>R3</td>
<td>5.69</td>
<td>4.14</td>
<td>T3</td>
<td>5.96</td>
<td>818</td>
</tr>
<tr>
<td>R4</td>
<td>5.34</td>
<td>4.51</td>
<td>T4</td>
<td>5.74</td>
<td>846</td>
</tr>
<tr>
<td>R5</td>
<td>5.59</td>
<td>4.18</td>
<td>T5</td>
<td>5.96</td>
<td>964</td>
</tr>
<tr>
<td>R6</td>
<td>5.48</td>
<td>3.64</td>
<td>T6</td>
<td>6.06</td>
<td>885</td>
</tr>
<tr>
<td>R7</td>
<td>5.14</td>
<td>6.00</td>
<td>T7</td>
<td>5.62</td>
<td>771</td>
</tr>
<tr>
<td>R8</td>
<td>5.42</td>
<td>3.92</td>
<td>T8</td>
<td>5.64</td>
<td>871</td>
</tr>
<tr>
<td>R9</td>
<td>5.42</td>
<td>4.19</td>
<td>T9</td>
<td>6.04</td>
<td>850</td>
</tr>
<tr>
<td>R10</td>
<td>5.47</td>
<td>3.99</td>
<td>T10</td>
<td>6.05</td>
<td>919</td>
</tr>
</tbody>
</table>

To illustrate the experimental procedure, we will give the raw data only for the 8.7 kA discharge; the data for the other discharge are quite similar. In fig. 5 we show the main parameters of the discharge; namely, loop voltage, plasma current, radial position of the plasma column, hard X-ray emission, and currents in the vertical field and ohmic heating coils. The radial position of the plasma column is globally measured by sensing coils outside the vacuum chamber. The signals detected by the pick-up coils are shown in fig. 6 for the azimuthal and in fig. 7 for the radial coils. We note that whereas the signals of the azimuthal coils very closely resemble the signal of the total plasma current, as they should, the signals of the radial coils seem very distorted, due to the ripple of the toroidal magnetic field, as previously mentioned. This can be seen by the signals detected by the radial coils when only the toroidal coils are pulsed, without plasma. These signals are also shown in fig. 7.

Using the values of the poloidal magnetic field measured by the
Fig. 5 - Main parameters of the 8.7 kA discharge: loop voltage (a), radial position (b), plasma current (c), hard X-ray emission (d), current in ohmic heating transformer (e), and current in the vertical field coils (f).

Fig. 6 - Field measured by the tangential field coils in the 8.7 kA discharge. The labels T1 to T10 correspond to the coils whose positions are indicated in fig. 3b.
pick-up coils, we can determine the plasma cross section and other equilibrium quantities, as discussed in section 2. The time evolution of the shape of the plasma cross section is shown in figs. 8 and 9 for 8.7 and 10 kA discharges, respectively. In the former case, the plasma column is first displaced towards the outside of the torus and then towards the inside. For the latter, the plasma column is continuously displaced towards the inside of the torus. In both cases, however, the plasma column shows a downward displacement and shrinks in minor radius towards the end of the discharge. The radial displacement of the plasma column can be more quantitatively seen by plotting the position of the geometric center of the plasma filaments as a function of time. This is shown in fig. 10. In the same picture we also show the position of the plasma column as measured by the radial position coils. It is clear that although the two measurements qualitatively agree, there is a constant shift between the two results. This is probably due to a small asymmetry in the radial position coils. If we correct for this asymmetry by making...
Fig. 8 - Time evolution of the cross section of the plasma column in the 8.7 kA discharge. Four filaments were used to simulate the plasma current. The x denotes the position of the geometric center of the filaments. The symmetry axis of the torus is to the left of the figures. Time is computed from the beginning of the discharge.

a uniform shift in the results of these coils, the two sets of measurements also quantitatively agree.

The downward displacement of the plasma column can be seen by plotting the vertical position of the geometric center of its cross section as a function of time. This is shown in fig. 11. We see that the plasma column is almost one centimeter downward displaced at the
Fig. 9 - Time evolution of the cross section of the plasma column in the 10 kA discharge. Four filaments were used to simulate the plasma current. The x denotes the position of the geometric center of the filaments. The symmetry axis of the torus is to the left of the figures. Time is computed from the beginning of the discharge. The small indentation in the plasma cross section at t = 5 ms is probably not a real feature but a numerical inaccuracy due to the closeness of the plasma filaments.

end of the discharge. Considering that the initial minor radius of the plasma column is only 8 cm, this displacement is quite substantial. Furthermore, it makes the hot plasma strongly interact with the current limiter, producing the copious hard X-ray emission shown in fig. 5 at the end of the discharge. The compression of the plasma column in both minor and major radii is probably due to the value of the external vertical field becoming larger than necessary. The downward displacement, on the other hand, is probably due to an asymmetry in the vertical field coils or incomplete cancellation of the vertical stray field produced by the toroidal field coils. This downward displacement has also been measured by Ueta using sensing coils outside the vacuum chamber\textsuperscript{23}.

Because the total plasma current is kept approximately constant and the cross section of the plasma column diminishes during the
Fig. 10 - Radial position of the geometric center of the plasma filaments \((e)\) as a function of time for the 8.7 kA (a) and 10 kA (b) discharges. The open circles represent the center of the plasma column determined from the radial position coils\(^2\).

Fig. 11 - Vertical position of the geometric center of the plasma cross section as a function of time for the 8.7 kA (a) and 10 kA (b) discharges.

discharge, the current density profile has to become steeper, increasing the \textit{internal} inductance \(R_i\) (eq. (3)). This is seen in fig. 12 where the parameter \(\Lambda\) (eq. (4)) is plotted as a function of time. \textbf{Clearly}, the value of \(\Lambda\) can also increase due to an \textit{increase} in the value of \(\beta_p\), caused by compressional heating by the vertical magnetic field. The maximum possible heating occurs under the condition of \textit{adiabatic} compression. In this case, conservation of \textit{poloidal} and toroidal magnetic
We thank Dr. A. Wooton for useful discussions. This work has

suggested profiles for the pressure and current density in the plasma,

which is much simpler than the free-boundary one. The dynamic equilibrium code can solve the fixed boundary equilibrium problem plasma current, and the values of \( A \) and \( B \) one can use a magnetohydrodynamic in tokamak discharges, knowing the shape of the cross section, the total section of the plasma column and other relevant equilibrium quantities can provide an accurate and fast determination of the shape of the cross section. We conclude that the diagnostic system described in this paper
decrease. However, for our conditions, it can be easily shown that for \( a > b \), the value of \( a \) is almost independent of the value of \( b \), whereas the value of \( b \) depends on the values of \( a \) and \( A \), where \( A \) is the average minor radius of the plasma cross section, whereas the value of \( b \) is set by the scale of the plasma.

\[
d_R = \frac{a}{\sqrt{1 + A - \frac{a}{b}}}
\]

Fig. 12 - Time evolution (a) of the equation (b) and (c) discharges.

- Fig. 12 - Time evolution (a) of the equation (b) and (c) discharges.
REFERENCES


Resumo

Um sistema de diagnóstico para determinar a forma da secção transversal da coluna de plasma no tokamak TBR-1 é descrito. O sistema é baseado numa técnica desenvolvida por Swain e Neilson e utiliza medidas das componentes radial e azimutal do campo magnético na entorno da coluna de plasma. Com este sistema foi verificado que gas normais a coluna de plasma apresenta um deslocamento sistemático para baixo do plano equatorial da máquina e encolhe tanto em relação ao raio maior como ao raio menor.