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Anisotropic Universe with Non-Linear Photons

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Abstract The metric and potential for a cosmological solution of the equations for the gravitational and electrical fields arededuced from a non-minimal coupled lagrangean in the vacuum, ultra-relativistic and dust filled Universe. The stability of the solution is also investigated.

1. INTRODUCTION

In the study of the modalities of non-minimal coupling between gravity and electromagnetism, Novello and Salim[1] have deduced the field equations from the lagrangean

$$L = \frac{\sqrt{-g}}{16\pi}\left[\frac{c^4 R}{G}(1 + \lambda A_\mu A^\mu) - F_{\mu\nu}F^{\mu\nu}\right] + L_{\text{matter}} \tag{1}$$

By variation of $g^{\mu\nu}$ in (1), one gets the equations

$$(1 + \lambda A^2)G_{\mu\nu} + \lambda A_\mu A_\nu R - \lambda \Box A^2 g_{\mu\nu} + \lambda A^2{}_{|\mu||\nu}$$
$$= -\frac{8\pi G}{c^2}(E_{\mu\nu} + T_{\mu\nu}) \tag{2}$$

with notation defined at the end of this article.

By variation of A_μ in eq. (1), one gets

$$F^{\mu\nu}{}_{||\nu} = \frac{c^4\lambda}{2G} R A^\mu + \frac{4\pi}{c} J^\mu \tag{3}$$

where J^μ is thecurrent term, derived from thematter part of the lagrangean.

The system of equations *(2)* and (3) may be solved in many specific situations and the most interesting may be the cosmological one, seeking the possibility to detect anisotropies in the Universe.

In general, the search for alternative equations to Einstein's equations for the gravitational field and other fields coupledwiththe latter comes in response to the necessity to avoid the presenceofthe initial cosmological singularity predicted by Hawking and Penrose's theorems[2], based on the validity of Einstein's equations.

In this study the author presents an exact and stable solution to the system of equations (2) and (3) for the anisotropic cosmological case.

2. FORMULATION OF THE COSMOLOGICAL EQUATION

The Universe will be considered filled with a fluid described by the momentum-energy tensor

$$T_{\mu\nu} = \left(\frac{p}{c^2} + \rho\right) V_\mu V_\nu - \frac{p}{c^2}\, g_{\mu\nu} \tag{4}$$

whose trace is $T = \rho - 3p/c^2$.

In the Universe as a whole one can consider that, despite local fluctuations, the electromagnetic field vanishes, so that

$$F_{\mu\nu} = 0 \tag{6}$$

whence

$$E_{\mu\nu} = 0 \tag{7}$$

Supposing $J^\mu = 0$, eq. (3) yields

$$R = 0 \tag{8}$$

Taking the trace of eq. (2), one gets

$$R + 3\lambda \Box A^2 = \frac{8\pi G}{c^2}\, T \tag{9}$$

which, according to eq. (8), reads

$$\lambda \Box A^2 = \frac{8\pi G}{3c^2}\, T \tag{10}$$

Defining

$$\Omega \equiv 1 + \lambda A^2 \tag{11}$$

eq. (2) becomes

$$\Omega R_{\mu\nu} + \Omega_{|\mu||\nu} = -\frac{8\pi G}{c^2}\left(T_{\mu\nu} - \frac{1}{3}\, T g_{\mu\nu}\right) \tag{12}$$

Using the relations (4) and (5), eq.(12) becomes

$$\Omega R_{\mu\nu} + \Omega_{|\mu||\nu} = + \frac{8\pi G}{c^2}\left\{\frac{\rho g_{\mu\nu}}{3} - \left(\rho - \frac{p}{c^2}\right) V_\mu V_\nu\right\} \tag{13}$$

with the condition, derived from eq. (10)

$$\Box\Omega = \frac{8\pi G}{3c^2} T \tag{14}$$

In this paper the author intends to solve the system of eqs. (13) and (14) for the anisotropic cosmic geometry.

3. THE ANISOTROPIC METRIC

Tlie slmplest metric for a Universe in which theexpansion rate changes with the dlrection is[3]

$$ds^2 = c^2dt^2 - A^2(ct)dx^2 - B^2(ct)dy^2 - C^2(ct)dz^2 \tag{15}$$

In rectangular comoving coordinates.

The non zero components of the Ricci curvature tensor, for thls metric:, are

$$R^0{}_0 = \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} \tag{16.a}$$

$$R^1{}_1 = \frac{\ddot{A}}{A} + \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) \tag{16.b}$$

$$R^2{}_2 = \frac{\ddot{B}}{B} + \frac{\dot{B}}{B}\left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right) \tag{16.c}$$

and

$$R^3{}_3 = \frac{\ddot{C}}{C} + \frac{\dot{C}}{C}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) \tag{16.d}$$

The covarlant derlvatives of Ω become

$$\Omega^{|0}{}_{||0} = \ddot{\Omega} \tag{17.a}$$

$$\Omega^{|1}{}_{||1} = \frac{\dot{A}\dot{\Omega}}{A} \tag{17.b}$$

$$\Omega^{|2}{}_{||2} = \frac{\dot{B}\dot{\Omega}}{B} \tag{17.c}$$

and

$$\Omega^{|3}{}_{||3} = \frac{\dot{C}\dot{\Omega}}{C} \tag{17.d}$$

Since in comoving coordinates $V^{\mu} = \delta^{\mu}{}_{0}$, eqs. (13) and (14), in the metric of eq.(15), become

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\ddot{\Omega}}{\Omega} = -\frac{8\pi G}{c^2\Omega}\left(\frac{2}{3}\rho + \frac{p}{c^2}\right) \tag{18.a}$$

$$\frac{\ddot{A}}{A} + \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C} + \frac{\dot{\Omega}}{\Omega}\right) = \frac{8\pi G}{3c^2\Omega}\rho \tag{18.b}$$

$$\frac{\ddot{B}}{B} + \frac{\dot{B}}{B}\left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C} + \frac{\dot{\Omega}}{\Omega}\right) = \frac{8\pi G}{3c^2\Omega}\rho \tag{18.c}$$

$$\frac{\ddot{C}}{C} + \frac{\dot{C}}{C}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{\Omega}}{\Omega}\right) = \frac{8\pi G}{3c^2\Omega}\rho \tag{18.d}$$

$$\frac{\ddot{\Omega}}{\Omega} + \frac{\dot{\Omega}}{\Omega}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = \frac{8\pi G}{c^2\Omega}\left(\frac{1}{3}\rho - \frac{p}{c^2}\right) \tag{18.e}$$

Defining

$$X \equiv \frac{\dot{A}}{A}, \quad Y \equiv \frac{\dot{B}}{B}, \quad Z \equiv \frac{\dot{C}}{C}, \quad W \equiv \frac{\dot{\Omega}}{\Omega}$$
$$k \equiv \frac{8\pi G}{c^2}; \quad Q \equiv \frac{\rho}{\Omega} \quad \text{and} \quad P \equiv \frac{p}{c^2\Omega} \tag{19}$$

eqs. (18) become

$$\dot{X} + \dot{Y} + \dot{Z} + \dot{W} + X^2 + Y^2 + Z^2 + W^2 = k\left(-\frac{2}{3}Q - P\right) \tag{20.a}$$

$$\dot{X} + X(X + Y + Z + W) = kQ/3 \tag{20.b}$$

$$\dot{Y} + Y(X + Y + Z + W) = kQ/3 \tag{20.c}$$

$$\dot{Z} + Z(X + Y + Z + W) = kQ/3 \tag{20.d}$$

$$\dot{W} + W(X + Y + Z + W) = k\left(\frac{1}{3}Q - P\right) \tag{20.e}$$

This set of five equations in the six unknowns X, Y, Z, W, Q and P requires a sixth equation for a unique solution, which is just the "state equation" obeyed by the cosmological fluid

$$p = p(\rho) \qquad \text{or} \qquad P = P(Q) \tag{21}$$

4. ANISOTROPIC CASE IN VACUUM

If $p = \rho = 0$ then $P = Q = 0$ and from eqs (20) one concludes that[4]

$$\frac{\dot{X}}{X} = \frac{\dot{Y}}{Y} = \frac{\dot{Z}}{Z} = \frac{\dot{W}}{W} = -(X + Y + Z + W) \tag{22}$$

Therefore, one can wrlte X, Y and Z as

$$X = \xi W, \quad Y = \eta W \quad \text{and} \quad Z = \zeta W \tag{23}$$

Then, eq. (20.e) becomes

$$\dot{W} = -KW^2 \tag{24}$$

where

$$K = \xi + \eta + \zeta + 1 \tag{25}$$

The solution of eq. (24) is

$$\frac{1}{Kt+C} \tag{26}$$

Hence X, Y, and Z are given by

$$X = \frac{\xi}{Kt+C}, \quad Y = \frac{\eta}{Kt+C}, \quad Z = \frac{\zeta}{Kt+C} \tag{27}$$

Carrying these results into eqs. (19) and integrating, one gets

$$\Omega = \left(\frac{t}{t_0}\right)^{1/K} \tag{28.a}$$

$$A = \left(\frac{t}{t_0}\right)^{\xi/K} \tag{28.b}$$

$$B = \left(\frac{t}{t_0}\right)^{\eta/K} \tag{28.c}$$

$$C = \left(\frac{t}{t_0}\right)^{\zeta/K} \tag{28.d}$$

where the integration constants are incorporated as a translation in the time axis.

Identifying the parameters

$$\alpha \equiv \frac{\xi}{K}, \quad \beta \equiv \frac{\eta}{K}, \quad \gamma \equiv \frac{\zeta}{K} \quad \text{and} \quad \omega \equiv \frac{1}{K} \tag{29}$$

one sees that

$$\alpha + \beta + \gamma + \omega = 1 \tag{30}$$

Moreover, taking relations (23) into eq. (20.a), with $Q=P=0$, and taklng tnto account eq. (24), one sees that

$$a^2 + \beta^2 + \gamma^2 + \omega^2 = 1 \tag{31}$$

The solution of the non-minimal ocupling equations in the anlsotropic cosmological case for the vacuum is, therefore, the metric

$$ds^2 = c^2dt^2 - \left(\frac{t}{t_0}\right)^{\alpha} dx^2 - \left(\frac{t}{t_0}\right)^{\beta} dy^2 - \left(\frac{t}{t_0}\right)^{\gamma} dz^2 \tag{32}$$

together with the four-potential given by

$$R = 1 + \lambda A^2 = \left(\frac{t}{t_0}\right)^{u} \tag{33}$$

where a, β, γ and ω obey the relations (30) and (31).

The volume in this Unlverse is given by

$$V = \iiint \sqrt{|\det g_{ij}|}\ dxdydz$$

$$= \left(\frac{t}{t_0}\right)^{\alpha+\beta+\gamma} \iiint dxdydz = \left(\frac{t}{t_0}\right)^{1-\omega} \iiint dxdydz \tag{34}$$

The rate of change in the voiurne is then

$$\frac{\dot{V}}{V} = \frac{1-\omega}{(t/t_0)} \tag{35}$$

The solution (32) is similar to the Kasner[5] solution in which the conditions (30) and (31) are merely

$$\alpha + \beta + \gamma = \alpha^2 + \beta^2 + \gamma^2 = 1 \tag{36}$$

Belinsky and Khalatnikov[6] have shown that this case can be thought of as a Kasner solution for Einstein's theory in a five-dimentional empty space, the factor $\Omega(t)$ playing a role of the expansion factor of the flfth dimention.

5. STUDY OF THE STABILITY OF THE SOLUTION

The system of equations (20), in the case $Q = P = 0$, is an autonomous systern in four dimensions, with the first of them justplaying a role of a constraint, slnce one can write it, taking the others into account, as

$$XY + XZ + XW + YZ + YW + ZW = 0 \tag{37}$$

Taking X as representative of the spatial variables, one can exbihit the: possible solutions of the system in a two-dimentional diagram. As seen in Fig.1, the solutlons are straight lines passing bythe orlgln as one can see from eqs. (23).

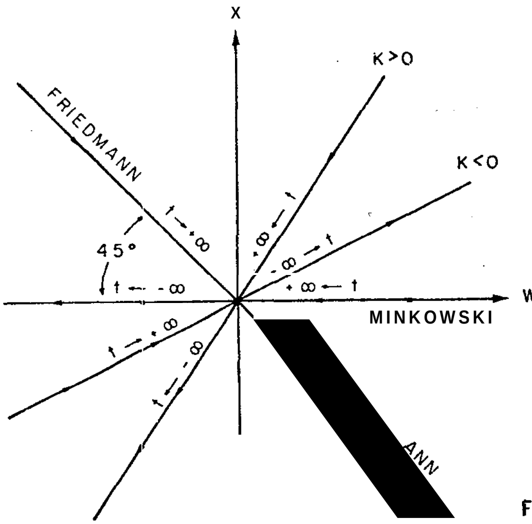

Fig.1 - Cosmological Solutions
the case $p = 0$

The origin is a singular point of the system, that is, in the origin, dX/dW is indeterminated, since $\dot{X} = \dot{W} = 0$. It represents the values of X and Wwhen the parameter t tends to $\pm\infty$. Thus the origin is a *stellar node*[7] of the system. The system will be stable if the lines tend to the origin for increasing values of t, and will be unstable otherwise.

As. can be seen from Fig.1, the system is *stable*.

A particular interestlng case is the one where A = B = C, that is the Friedmann isotropic Universe. In this case, eqs. (30) and (31), give

$$\begin{aligned} 3\alpha + \omega &= 1 \\ 3\alpha^2 + \omega^2 &= 1 \end{aligned} \tag{38}$$

The solution of thls system is either

$$\alpha = 0 \quad , \omega = 1 \tag{39}$$

or

$$\alpha = 1/2 \ , \ \omega = -1/2 \tag{40}$$

The case of eq. (39) corresponds to the Minkowski space and eq. *(40)* corresponds to the Friedmann space, that is, respectively to the metrics

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 \tag{41}$$

and

$$ds^2 = c^2dt^2 - \left(\frac{t}{t_0}\right)^{1/2} (dx^2 + dy^2 + dz^2) \tag{42}$$

with the respective potentials

$$\Omega = 1 + \lambda A^2 = \left(\frac{t}{t_0}\right) \tag{43}$$

and

$$R = 1 + \lambda A^2 = \left(\frac{t}{t_0}\right)^{1/2} \tag{44}$$

Note that the Fríedmann metric *(42)* corresponds to an euclidean spatial section.

In terms of X, Y, Z and W, one has

Minkowski case $\quad X = Y = Z = 0$ (45)
$\qquad W = 1/t$

Friedmann case $\quad X = Y = Z = 1/2t$ (46)
$\qquad W = - 1/2t$

6. THE CASE OF ULTRA-RELATIVISTIC PARTICLES

If, instead of $p = \rho = 0$, one has

$$p = \frac{c^2\rho}{3} \quad \text{or} \quad P = \frac{Q}{3} \tag{47}$$

which means that if the Universe is filled with an ultra-relativistic particle gas (neutrinos, for example), besides the non-linear photons originated from the non-minimal coupling, then the set of eqs. (20) becomes

$$\dot{X} + \dot{Y} + \dot{Z} + \dot{W} + X^2 + Y^2 + Z^2 + W^2 = kQ \tag{48.a}$$

$$\dot{X} + X(X + Y + Z + W) = kQ/3 \tag{48.b}$$

$$\dot{Y} + Y(X + Y + Z + W) = kQ/3 \tag{48.c}$$

$$\dot{Z} + Z(X + Y + Z + W) = kQ/3 \tag{48.d}$$

$$\dot{W} + W(X + Y + Z + W) = 0 \qquad (48.e)$$

If one tries a solution like eqs. (26) and (27) one sees that this implies $Q \equiv 0$.

On the other hand, eqs. (48) yield

$$\frac{\dot{X}}{X} - \frac{kQ}{3X} = \frac{\dot{Y}}{Y} - \frac{kQ}{3Y} = \frac{\dot{Z}}{Z} - \frac{kQ}{3Z} = \frac{\dot{W}}{W} = X + Y + Z + W \qquad (49)$$

which gives the set

$$\frac{\dot{W}}{W} = \frac{\dot{X} - \dot{Y}}{X - Y} = \frac{\dot{Y} - \dot{Z}}{Y - Z} = \frac{\dot{Z} - \dot{X}}{Z - X} \qquad (50)$$

From eq. (50) one concludes that

$$\alpha W = X - Y \text{ , } \beta W = Y - Z \text{ and } \gamma W = Z - X \qquad (51)$$

with

$$\alpha + \beta + \gamma = 0 \qquad (52)$$

It is possible to choose the solution

$$X = Y = Z \text{ and } W = 0 \qquad (53)$$

Then eqs. (48.a) and (48.b) yield

$$3\dot{X} + 3X^2 = - kQ \qquad (54.a)$$

$$\dot{X} + 3X^2 = kQ/3 \qquad (54.b)$$

which lead to

$$X = - 2X^2 \qquad (55)$$

whose solution is

$$X = \frac{1}{2t + C} = Y = Z \qquad (56)$$

Then one has, according to equation (54.b)

$$kQ = \frac{3}{(2t + C)^2} \qquad (57)$$

The result (56) yields a Friedmann-like metric just as eq. (42), with the same time-dependence. But now there is not a time-dependent electric potential because once $W=0$, then $\Omega=1+\lambda A^2$=constant. The energy density and ultra-relativistic particle gas pressure, however, obey a t^{-2} time-dependence.

With regard to the stability one can see that the system (48) is also an autonomous system with eq. (48.a) palying the role of a constraint. Once again the origin is a "stable stellar node" of the system.

7. THE DUST-FILLED UNIVERSE

In the case of a dust-filled universe one has

$$p = P = 0 \tag{58}$$

and the set of eqs. (20) becomes

$$\dot{X} + \dot{Y} + \dot{Z} + \dot{W} + X^2 + Y^2 + Z^2 + W^2 = -2kQ/3 \tag{59.a}$$

$$\dot{X} + X(X + Y + Z + W) = kQ/3 \tag{59.b}$$

$$\dot{Y} + Y(X + Y + Z + W) = kQ/3 \tag{59.c}$$

$$\dot{Z} + Z(X + Y + Z + W) = kQ/3 \tag{59.d}$$

$$\dot{W} + W(X + Y + Z + W) = kQ/3 \tag{59.e}$$

Once again the Kasner-like solution yields the vanishing of Q and then of p.

Assuming again that

$$X = \xi W, \quad Y = \eta W \quad \text{and } Z = \zeta W \tag{23}$$

eq.(59.e) is written

$$\dot{W} + KW^2 = kQ/3 \tag{60}$$

where

$$K = \xi + \eta + \zeta + 1 \tag{25}$$

But if one takes eq.(23) into eqs. (59b, c, d) one sees that

$$\xi = \eta = \zeta = 1 \tag{61}$$

Consequently $K = 4$ and eq. (60) reads

$$\dot{W} + 4W^2 = kQ/3 \tag{62}$$

Equation (59.a) then becomes

$$4\dot{W} + 4W^2 = -2kQ/3 \tag{63}$$

Eqs. (62) and (63) together give

$$\dot{W} + 2W^2 = 0 \qquad (64)$$

whose solution is

$$W = \frac{1}{2t + C} = X = Y = Z \qquad (65)$$

Carrying this result into eq. (62), one obtains

$$kQ = \frac{6}{(2t + C)^2} \qquad (66)$$

Once again the result is a Friedmann-like Universe, now with an electric potential with the same time-dependence as the metric.

The same considerations made in the previous case in regard to stability should be also made here.

8. SUMMARY AND CONCLUSIONS

The three possibilities considered for the equation of state furnished the following results.

i) $p = \rho = 0$

$$ds^2 = c^2dt^2 - \left(\frac{t}{t_0}\right)^{\alpha} dx^2 - \left(\frac{t}{t_\beta}\right)^{\beta} dy^2 - \left(\frac{t}{t_n}\right)^{\gamma} dz^2$$

and

$$1 + \lambda A^2 = \left(\frac{t}{t_0}\right)^{\omega}$$

with

$$\alpha + \beta + \gamma + \omega = \alpha^2 + \beta^2 + \gamma^2 + \omega^2 = 1$$

Particular cases are

Friedmann: $\alpha = \beta = \gamma = 1/2$

$\omega = -1/2$

and

Minkowski: $\alpha = \beta = \gamma = 0$

$\omega = 1$

ii) $p = \dfrac{c^2\rho}{3} \propto t^{-2}$

$$ds^2 = c^2dt^2 - \left(\frac{t}{t_0}\right)^{1/2} (dx^2 + dy^2 - dz^2)$$

and

$$1 + \lambda A^2 = \text{constant}.$$

iii) $p = 0, \quad \rho \propto t^{-2}$

$$ds^2 = c^2 dt^2 - \left(\frac{t}{t_0}\right)^{1/2} (dx^2 + dy^2 + dz^2)$$

and

$$1 + \lambda A^2 = \left(\frac{t}{t_0}\right)^{1/2}$$

As demonstrated before, the presence of an energetical or material content in the Universe, besides the non-linear photons, removes the anisotropy which may be present in the vacuum case.

A possibility for the history of the Universe in this non-minimal coupling model is that in the primeval era the anisotropy arises but subsequently, in the radiation era, the energy content leads to isotropy which is actually present in the matter era. The radiation (neutrino) and the matter may have arisen from the fluctuations of the primeval vacuum, carrying a matter-antimatter pair formation. As we have shown, the solutions are stable.

The author would like to aknowledge Dr. M. Novello, frorn CBPF, for supervising the thesis which led to the development of this paper.

NOTATION

$A_\mu \equiv (\phi, \vec{A})$ - four-vector potential

$E_{\mu\nu} \equiv \frac{1}{4\pi c^2} (F_{\mu\alpha} F^{\alpha}_{\nu} + \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} g_{\mu\nu})$ - momentum energy tensor of the electromagnetic field

$F_{\mu\nu} \equiv A_{\mu|\nu} - A_{\nu|\mu}$ - electromagnetic tensor

G - gravitational constant

$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ - Einstein tensor

g $\equiv$ det $(g_{\mu\nu})$

$g_{\mu\nu}$ - metric tensor

$J^{\mu} \equiv (c\rho, \vec{j})$ - current density four-vector

p - isotropic pressure

$R^{\alpha}{}_{\eta\beta\gamma} \equiv \Gamma^{\alpha}{}_{\beta\eta|\gamma} - \Gamma^{\alpha}{}_{\eta\gamma|\beta} + \Gamma^{\alpha}{}_{\tau\gamma}\Gamma^{\tau}{}_{\beta\eta} - \Gamma^{\alpha}{}_{\tau\beta}\quad{}_{\gamma\eta}$ - Riemann curvature tensor

$R_{\mu\nu} \equiv R^{\alpha}{}_{\mu\alpha\nu}$ - Ricci tensor

$R \equiv g^{\mu\nu}R_{\mu\nu}$ - curvature scalar

$T_{\mu\nu} \equiv \frac{2}{c^2\sqrt{-g}} \frac{\delta L_{\text{matter}}}{\delta g_{\mu\nu}}$ - momentum energy tensor of the matter

ρ - charge density

$\Box\phi \equiv g^{\mu\nu}\phi_{|\mu||\nu}$ - d'Alembertian operator

$f_{|\nu} \equiv \frac{\partial f}{\partial x^{\nu}}$ - partial derivative

$V^{\mu}{}_{||\lambda} \equiv V^{\mu}{}_{|\lambda} + \Gamma^{\mu}{}_{\lambda\kappa}V^{\kappa}$ - covariant derivative

$\bullet \equiv \frac{\partial}{\partial t}$ - time derivative

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Resumo

São apresentados a métrica e o potencial para uma solução cosmológica das equações dos campos gravitacional e eletromagnético deduzidos de uma lagrangeana com acoplamento não mínimo para o vácuo, um Universo com partículas ultra-relativísticas e com poeira. A estabilidade da solução é também investigada.