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Three-Body Problem with Bargmann Potentials*

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The three-body bound state is examined using a family of Bargmann potentials which give exactly the same s-wave phase shift and the binding energy for the two-body system. The binding energy of the three-body system is found to vary over a wide range among the potentials. This is similar to the off-shell effect illustrated by Fiedeldey by using nonlocal separable potentials of rank two.

Estuda-se o estado ligado, de um sistema a três corpos, fazendo-se uso de uma família de potenciais de Bargmann que produzem exatamente a mesma defasagem de onda s, como também a energia de ligação para o sistema a dois corpos. Verifica-se que a energia de ligação, do sistema a três corpos, varia consideravelmente entre os potenciais da família, o que é semelhante ao que ocorre fora da camada de energia conforme ilustrado por Fiedeldey com potenciais não locais, separãveís, de posto 2.

1. INTRODUCTION

It is well known that two potentials which are equivalent to each other, with respect to the relevant two-body system, can in general yield dif-

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ferent results for the three-body or many-body system. Here, by equivalent potentials we mean ones which give identical scattering phaseshifts and the binding energies of bound states, if any. Some time ago, Fiedeldey (1969) illustrated this by means of nonlocal separable potentials. He constructed a family of equivalent potentials for the two-nucleon system in the s-state. He then showed that the triton binding energy with these potentials varied from 7.5 MeV to about 16 MeV.

Since nonlocal separable potentials can exhibit features which are uncommon for local potentials [see e.g. Leung and Park (1969)], one might wonder whether or not the very large variation of the triton binding energy found by Fiedeldey is peculiar to nonlocal separable potentials. The purpose of this note is to show, by using a family of Bargmann potentials, that a similar situation can occur also with local potentials.

2. A FAMILY OF BARGMANN POTENTIALS

We start with the standard Bargmann potential (Newton 1966)

$$V(r) = -\frac{8b^2\beta e^{-2br}}{\left[1+\beta e^{-2br}\right]^2} , \qquad (1)$$

where b and β are related to another parameter $x > 0$ by $\beta = (b+x)/(b-x)$. We use the same notations as those of Newton (1966).

Potential (1) leads to the s-wave phase shift δ given by

$$k \cot \delta = \frac{-bx+k^2}{b+x} , \qquad (2)$$

and a bound state with the binding energy $x^2/2\mu$. We consider particles of the same (nucleon) mass m, hence $2\mu = m$.

Now, there is a one-parameter family of potentials that produce the same phase shift and the same binding energy. They are

$$V_c(r) = -4x \frac{d}{dr} \left[\frac{g_c(x,r)\sinh br}{g_c(x+b,r)-g_c(x-b,r)} \right], \qquad (3)$$

where $g_c(x,r) = (e^{-xr} + c \sinh xr)/x$. For $c = 2$, $V_c(r)$ reduces to V of (1). The potential V_c has an asymptotic tail proportional to e^{-2xr} except when $c = 2$, in which unique case the tail is proportional to e^{-2br}.

For the neutron-proton system in the triplet s-state, the empirical values for the effective range and the deuteron binding energy are produced by the above potentials if $x^{-1} = 4.31$ fm and $b = 0.944$ fm^{-1}.

3. TRITON BINDING ENERGY

In the triton, the effective nucleon-nucleon interaction would be an average of the potentials for the singlet and triplet states. Therefore, we adjust x such that the empirical triton binding-energy, 8.48 MeV, is obtained when $c = 2$, while we keep the value of b fixed at 0.944 fm^{-1} so that the effective range remains approximately the same as that for the singlet s-state. We then vary c to see how the triton binding energy B_t is affected.

The three-body calculation has been done using the K-harmonics code, which has been developed and well-tested by Vallières et *al.* (1976). In the K-harmonics (or hyperspherical harmonics) method, the three-body wave function is expanded into the so-called grand partial waves, and the three-body Schrödinger equation is reduced to an infinite family of coupled ordinary differential equations which can be solved as accurately as desired. Hence, the results obtained are essentially exact. In Fig. 1 we show the triton binding energy versus c. The other parameters of the potential are $x = 0.055$ fm^{-1} and $b = 0.944$ fm^{-1}. Note that $B_t = 8.48$ MeV for $c = 2$. Fig. 2 shows the mean square radius $\langle r^2 \rangle$ versus c. It is clear that these quantities vary considerably for different values of c.

In closing, let us note that the photodisintegration cross section of the deuteron is also sensitive to c, and comparison with experiments

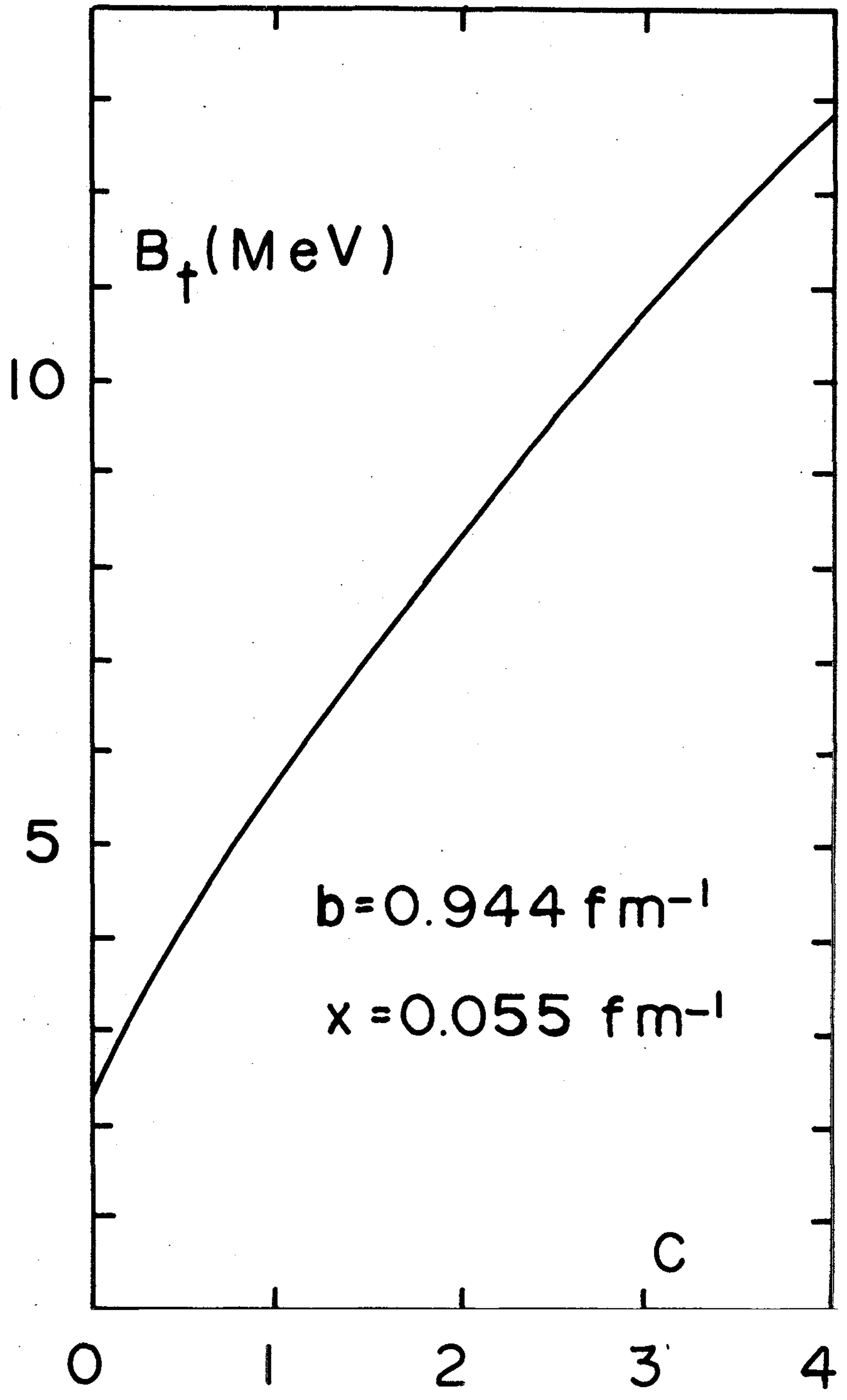

Fig.1 – The triton binding energy B_t (MeV) versus c.

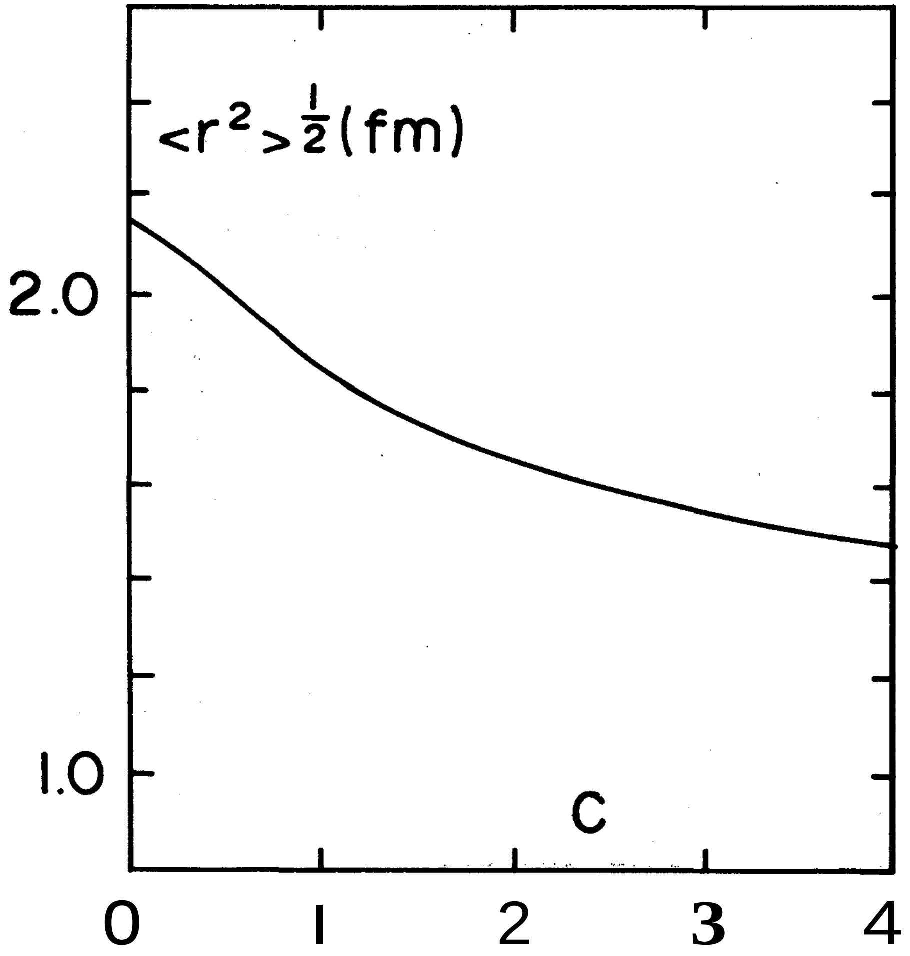

Fig.2 – The mean square radius $<r^2>^{1/2}$ of the triton versus c. The other parameters of the potential are as in Fig.1.

exclude V_c with c # 2 (Newton 1957; Levinger and Rustigi 1957). Also, as we noted above, V_c has a very long-range tail unless $c = 2$, and hence is not really acceptable as a model for the nucleon-nucleon potential. The same objection can be applied to Fiedeldey's potentials (Fiedeldey 1969).

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